Simplectic geometry and the canonical variables for Dirac-Nambu-Goto and Gauss-Bonnet system in string theory

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Abstract

Using a strongly covariant formalism given by Carter for the deformations dynamics of p-branes in a curved background and a covariant and gauge invariant geometric structure constructed on the corresponding Witten's phase space, we identify the canonical variables for Dirac-Nambu-Goto [DNG] and Gauss-Bonnet [GB] system in string theory. Future extensions of the present results are outlined.

I. INTRODUCTION

The interest in physical systems characterized by extended structures goes back to the XIX th Century and to Lord Kelvin's "aether atoms", for which a spatial extension was postulated in order to accommodate a complex structure which would be have both as an elastic solid (conveying the transverse wave motion of electromagnetism) and viscuss liquid (dragged by the earth in its orbital motion).

In the XX the Century, there have been three active motivations leading to either classical or quantum extendons. On the other hand, the physics of condensed matter (including biological systems) have revelated that membranes and two-dimensional layers play an important role; in some case, there also appear one-dimensional filaments (or strings). Similar structures appear in astrophysics and cosmology, one example being the physics of Black holes, in which the "membrane" is the boundary layer between the hole and the embedding spacetime, and another example is represented by the hypothetical cosmic strings.

In last years a considerable amount of effort has been devoted for developing a quantum field theory of such

extended objets (which in fact, will constitute the ultimate framework) for a complete M theory; however, it has not yet been fully developed. The problem is that the dynamics of extended objets is highly nonlinear and the standar methods are not directly applied. However, using a covariant canonical formalism introduced by Witten [1] in recent letters [2, 3, 4, 5] the basic elements to quantize extended objects (in particular bosonic p-branes) has been explore, for example, in [3] we established the bases to study the quantization aspects of p-branes with thickness, because, when adding it to the [DNG] action has an important effect in QCD [6], among other things. In [4] has been demonstrated that the presence of Gauss-Bonnet [GB] topological term in the [DNG] action describing strings, has a dramatic effect on the covariant phase space formulation of the theory, in this manner, we shall obtained a completely different quantum field theory. Recently, using the results given in [4] we identify the covariant canonical variables for [DNG] p-branes and [GB] strings, among other things [7]. However, we found a little problem, because, the canonical variables found for [DNG] are identified with spacetime indices, whereas, the canonical variables for [GB] strings with worldsheet indices, in this manner, if we add the [GB] term to the [DNG] action describing strings, we need identify the pullback on the canonical variables for [GB] strings in order to obtain it in terms of spacetime indices, and thus, to study in a covariant form the quantization aspects for [DNG-GB] system in string theory, but it was not clarified.

In this manner, the purpose of this article is to make, first, a generalization of the results presented in [3, 4] for a general Lagrangian constructed locally from the geometry of the worldvolume in a arbitrary background, after that, using a strongly covariant formalism given by Carter [8], we identify the canonical variables for [DNG-GB] system in string theory, in this manner, we resolve the problem found in [7].

This paper is organized as follows. In Sect.II, we make a generalization of the method utilized in [3, 4] for a Lagrangian constructed from the geometry of the worldvolume embedding in a arbitrary background, confirming as special case the results found in [3]. In Sect.III, we make an outline of the results found in [9], which, will be important the developed of this paper. In Sect. IV using a strongly covariant scheme of deformations introduced by Carter [8], we found the canonical variables for [GB] system in string theory, that unlike [7], the canonical variables of [GB] has spacetime indices, which will be determinant for the treatment of [DNG-GB] strings. In Sect. V with the results found in the previous sections we identify the canonical variables for [DNG-GB] system in string theory, and with this result, we clarified the problem that we found in [7]. In Sect. VI we give the conclusions and prospects.

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II. Symplectic potentials for p-branes in a curved background

In recent letters, a covariant and gauge invariant symplectic structure for [DNG] p-branes [2], for membranes with quadratic terms in the extrinsic curvature [3], and for the Gauss-Bonnet topological term propagating in a curved background [4] has been constructed. The form of constructing this geometric structure is by means of identifying from the arguments of the total divergences at the level of the Lagrangian a symplectic potential that does not contribute locally to the dynamics of the system, but its variation (the exterior derivative on the phase space) generates a geometric structure. In this manner, the purpose of this section is to generalize these results for a Lagrangian constructed from the geometry of the worldvolume embedding in a arbitrary spacetime.

For our aims, we will consider a local action depending on the embedding functions X^μ which is both invariant under worldvolume reparametrization and under rotations of the normals given by

$$S[X] = \int \sqrt{-\gamma} L d^D\xi, \tag{1}$$

where the Lagrangian L will be constructed locally from the geometry of the worldvolume as

$$L(\gamma^{ab}, K_{ab}{}^i, \widetilde{\nabla}_a K_{bc}{}^i), \tag{2}$$

here, γ^{ab}, $K_{ab}{}^i$ and $\widetilde{\nabla}_a$ is the metric induced, the extrinsic curvature and the covariant derivative under rotation of the normal vector field respectively [10].

Now, we need calculate the deformation of the Lagrangian given in equation (2) to identify the equations of motion and the symplectic potential for the theory described by the action (1). For this, we descompose an arbitrary infinitesimal deformation of the embedding δX^μ into its parts tangential and normal to the worldvolume, this is

$$\delta X^\mu = e^\mu{}_a \phi^a + n^\mu{}_i \phi^i, \tag{3}$$

where $n^\mu{}_i$ are the vector fields normal and $e^\mu{}_a$ are the vector field tangent to worldvolume, thus, the deformation operator is defined as

$$D = D_\delta + D_\Delta, \tag{4}$$

where

$$D_\delta = \delta^\mu D_\mu, \qquad \delta^\mu = n_i{}^\mu \phi^i, \tag{5}$$

and

$$D_\Delta = \Delta^\mu D_\mu, \qquad \Delta^\mu = e_a{}^\mu \phi^a, \tag{6}$$

in this manner, the variation of the equation (1) with the Lagrangian (2) is given by

$$\delta S = \int \sqrt{-\gamma} \nabla_a (L\phi^a) d^D\xi + \int \sqrt{-\gamma} [K^i \phi_i L + H^{ab}{}_i \widetilde{D}_\delta K_{ab}{}^i + H^{ab} D_\delta \gamma_{ab} + H^{abc}{}_i \widetilde{D}_\delta (\widetilde{\nabla}_a K_{bc}{}^i)] d^D\xi, \tag{7}$$

where

$$
\begin{aligned}
H_{ab} &= \frac{\partial L}{\partial \gamma_{ab}}, \\
H^{ab}{}_{i} &= \frac{\partial L}{\partial K_{ab}{}^{i}} = H^{ba}{}_{i}, \\
H^{abc}{}_{i} &= \frac{\partial L}{\partial \widetilde{\nabla}_a K_{bc}{}^{i}} = H^{acb}{}_{i}.
\end{aligned} \tag{8}
$$

On the other hand, using the deformation formalism introduced in [10] and writing the normal variation of γ_{ab}, $K_{ab}{}^{i}$ and $\widetilde{\nabla}_a K_{cb}{}^{i}$ in a curved background, we obtain

$$
\widetilde{D}_\delta K^i_{ab} = -\widetilde{\nabla}_a \widetilde{\nabla}_b \phi^i + K_{ac}{}^{i} K^{c}{}_{bj} \phi j + g(R(e_a, n_j)e_b, n^i), \tag{9}
$$

$$
\begin{aligned}
\widetilde{D}_\delta \widetilde{\nabla}_a K_{bc}{}^{i} &= \widetilde{\nabla}_a[-\widetilde{\nabla}_b \widetilde{\nabla}_c \phi^i + K_{db}{}^{i} K^{d}{}_{cj} \phi^j + g(R(e_b, n_j)e_c, n^i)\phi^j] - [\widetilde{\nabla}_b (K_a{}^{gj}\phi_j) + \widetilde{\nabla}_a (K_b{}^{gj}\phi_j) \\
&- \widetilde{\nabla}^g (K_{ba}{}^{j}\phi_j)] K_{gc}{}^{i} + [-\widetilde{\nabla}_c (K_a{}^{gj}\phi_j) - \widetilde{\nabla}_a (K_c{}^{gj}\phi_j) + \widetilde{\nabla}^g (K_{ca}{}^{j}\phi_j)] K_{gb}{}^{i} + [K_{ad}{}^{i} \widetilde{\nabla}^d \phi^j \\
&- K_{ad}{}^{j} \widetilde{\nabla}^d \phi^i - g(R(n_k, e_a)n^j, n^i)\phi^k] K_{bcj},
\end{aligned} \tag{10}
$$

with $g(R(e_a, n_j)e^a, n^i) = R_{\alpha\beta\mu\nu} n_j{}^{\alpha} e_a{}^{\beta} e^{a\mu} n^{i\nu}$, being $R_{\alpha\beta\mu\nu}$ the background Riemann tensor [3, 10]. Substituting Eqs. (9), (10) and removing the scalar field ϕ^i in (7) we obtain

$$
\begin{aligned}
\delta S &= \int \sqrt{-\gamma}[K^i L - 2K^{abi} H_{ab} - \widetilde{\nabla}_a \widetilde{\nabla}_b H^{ab}{}_{i} - K_{ac}{}^{i} K_b{}^{cj} H^{ab}{}_{j} + g(R(e_a, n^i)e_b, n^j)) H^{ab}{}_{j} + \widetilde{\nabla}_c \widetilde{\nabla}_b \widetilde{\nabla}_a H^{abci} \\
&+ 2K^{gi}_a \widetilde{\nabla}_b (H^{abc}{}_{j} K_{gc_j}) + 2K_b{}^{gi} \widetilde{\nabla}_a (H^{abc}{}_{j} K_{gc_j}) + K_{ba}{}^{i} \widetilde{\nabla}^g (H^{abc}{}_{j}) K_{gc}{}^{j} + K_a{}^{gi} \widetilde{\nabla}_a (H^{abc}{}_{j} K_{gb}{}^{j}) \\
&- K_{ca}{}^{i} \widetilde{\nabla}^g (H^{abc}_j K_{gb}{}^{j}) - \widetilde{\nabla}^d (H^{abc}{}_{j} K_{ad}{}^{j} K_{bc}{}^{i}) - \widetilde{\nabla}^d (H^{abci} K_{ad}{}^{j} K_{bcj}) - g(R(n^i, e_a)n^j, n^l) H^{abc}{}_{l} K_{bcj}]\phi_i \\
&+ \int \sqrt{-\gamma} \widetilde{\nabla}_a [L\phi^a - H^{ab}{}_{i} \widetilde{\nabla}_b \phi^i + \widetilde{\nabla}_b H^{ab}{}_{i} \phi^i - H^{abc}{}_{i} \widetilde{\nabla}_b \widetilde{\nabla}_c \phi^i + H^{abc}{}_{i} g(R(e_b, n_j)e_c, n^i)\phi^j + H^{abc}{}_{i} K_{db}{}^{i} K^{d}{}_{cj} \phi^j \\
&+ \widetilde{\nabla}_b H^{abc}{}_{i} \widetilde{\nabla}_c \phi^i - \widetilde{\nabla}_b \widetilde{\nabla}_c H^{cba}{}_{i} \phi^i - H^{bac}{}_{i} K_{gc}{}^{i} H_b{}^{gj} \phi_j - 2H^{abc}{}_{i} K_{gc}{}^{i} K_b{}^{gj} \phi_j - H^{cba}{}_{i} K_{gb}{}^{i} K_c{}^{gj} \phi_j \\
&+ H^{gbc}{}_{i} K_c{}^{ai} K_{bg}{}^{j} \phi_j + H^{gbc}{}_{i} K_b{}^{ai} K_{cg}{}^{j} \phi_j + H^{gbc}{}_{i} K_{bcj} K_g{}^{ai} \phi^j - H^{dbc}{}_{i} K_d{}^{aj} K_{bcj} \phi^i] d^D \xi,
\end{aligned} \tag{11}
$$

from last equation we can identify the equations of motion given by

$$
\begin{aligned}
K^i L \;& -2K^{ab^i} H_{ab} - \widetilde{\nabla}_a \widetilde{\nabla}_b H^{ab^i} - K_{ac}{}^{i} K_b{}^{cj} H^{ab}{}_{j} + g(R(e_a, n^i)e_b, n^j)) H^{ab}{}_{j} + \widetilde{\nabla}_c \widetilde{\nabla}_b \widetilde{\nabla}_a H^{abci} \\
+ \;& 2K_a{}^{gi} \widetilde{\nabla}_b (H^{abc}{}_{j} K_{gc_j}) + 2K_b{}^{gi} \widetilde{\nabla}_a (H^{abc}{}_{j} K_{gc_j}) + K_{ba}{}^{i} \widetilde{\nabla}^g (H^{abc}{}_{j}) K_{gc}{}^{j} + K_a{}^{gi} \widetilde{\nabla}_a (H^{abc}{}_{j} K_{gb}{}^{j}) \\
- \;& K_{ca}{}^{i} \widetilde{\nabla}^g (H^{abc}_j K_{gb}{}^{j}) - \widetilde{\nabla}^d (H^{abc}{}_{j} K_{ad}{}^{j} K_{bc}{}^{i}) - \widetilde{\nabla}^d (H^{abci} K_{ad}{}^{j} K_{bcj}) \\
- \;& g(R(n^i, e_a)n^j, n^l) H^{abc}{}_{l} K_{bcj} = 0,
\end{aligned} \tag{12}
$$

and we identify from the pure divergence term in (11)

$$
\begin{aligned}
\Psi^a &= \sqrt{-\gamma}[L\phi^a - H^{ab}{}_i\widetilde{\nabla}_b\phi^i + \widetilde{\nabla}_b H^{ab}{}_i\phi^i - H^{abc}{}_i\widetilde{\nabla}_b\widetilde{\nabla}_c\phi^i + H^{abc}{}_i g(R(e_b,n_j)e_c,n^i)\phi^j \\
&+ \widetilde{\nabla}_b H^{abc}{}_i\widetilde{\nabla}_c\phi^i - \widetilde{\nabla}_b\widetilde{\nabla}_c H^{cba}{}_i\phi^i - H^{bac}{}_i K_{gc}{}^i K_b{}^{gj}\phi_j - H^{abc}{}_i K_{gc}{}^i K_b{}^{gj}\phi_j - H^{cba}{}_i K_{gb}{}^i K_c{}^{gj}\phi_j \\
&+ 2H^{gbc}{}_i K_c{}^{ai} K_{bg}{}^j\phi_j + H^{gbc}{}_i K_{bcj} K_g{}^{ai}\phi^j - H^{dbc}{}_i K_d{}^{aj} K_{bcj}\phi^i],
\end{aligned}
\tag{13}
$$

as a symplectic potential for the theory described for a Lagrangian given in equation (2), that is ignored in the literature since that it does not contribute locally to the dynamics, but generates our geometrical structure on the phase space. Note that there exists a term involving explicitly the background curvature in Eq.(13).

Now, in the next lines we will take particular cases of the Lagrangian given in equation (2) using the previous results we will confirm the results given in [3]; for this, we take as first example the [DNG] p-branes action. As we know the [DNG] p-branes action is proportional to the area of the spacetime trajectory created by the brane, thus, if we take to $L = -\mu$, where μ is a constant characterizing the brane tension we have the well known action for [DNG] p-branes

$$
S = -\mu \int \sqrt{-\gamma} d^D\xi, \tag{14}
$$

in this manner, utilizing the Eqs. (8) we easily obtain

$$
\begin{aligned}
H_{ab} &= 0, \\
H^{ab}{}_i &= 0, \\
H^{abc}{}_i &= 0,
\end{aligned}
\tag{15}
$$

substituting the last result in the equation (12) we obtain

$$
K^i = 0, \tag{16}
$$

that corresponds to the equations of motion for [DNG] p-branes describing extremal surfaces [2, 3, 10].
On the other hand, if we consider the Eq. (15) in (13) we find

$$
\Psi^a = -\mu\sqrt{-\gamma}\phi^a, \tag{17}
$$

that corresponds to the symplectic potential for [DNG] p-branes. Thus, if we take the variation of Ψ^a (the exterior derivative on the phase space) given in (17) we will generate a geometrical structure on the phase space, to more details see [3].

As second example we will consider a Lagrangian that is quadratic in the extrinsic curvature, because of in many cases it was seen that [DNG] action is inadequate and there are missing corrective quadratic terms in the extrinsic curvature. For example, in the eighties Polyakov proposed a modification to the [DNG] action by adding a rigidity term constructed with the extrinsic curvature of the worldsheet generated by a string, and to include quadratic terms in the extrinsic curvature to the [DNG] action is absolutely necessary, because of its influence in the infrared region determines the phase structure of the string theory, in this manner, we can compute the critical behavior of random surfaces an their geometrical and physical characteristics [6]. In the treatment of topological defects [11], curvature terms are induced by considering an expansion in the thickness of the defect. Bosseau and Letelier have studied cosmic strings with arbitrary curvature corrections, finding for example, that the curvature correction may change the relation between the string energy density and the tension [12]. Furthermore, such models have been used to describe mechanical properties of lipid membranes [13]. Because of the above considerations, we will take a Lagrangian quadratic in the extrinsic curvature given by $L = \alpha K^i K_i$, here, α is a constant associated to the brane tension [3, 10]. Thus, if we substitute it in Eq. (8) we obtain

$$\begin{aligned} H_{ab} &= 2\alpha K^i K_{abi}, \\ H^{ab}{}_i &= 2\alpha \gamma^{ab} K_i, \\ H^{abc}{}_i &= 0. \end{aligned} \tag{18}$$

In this manner, in virtue to last equation the equation (8) takes the form

$$\widetilde{\triangle} K^i + \left(-g(R(e_a, n^j)e^a, n^i) + (\gamma^{ac}\gamma^{bd} - \frac{1}{2}\gamma^{ab}\gamma^{cd}) K_{ab}{}^j K_{cd}{}^i \right) K_j = 0, \tag{19}$$

that corresponds to the dynamics for the theory under study [3, 10].
In the same form, if we substitute the Eq. (18) into (13) we obtain

$$\Psi^a = 2\alpha\sqrt{-\gamma} \left[\frac{1}{2} K^j K_j \phi^a + \phi_i \widetilde{\nabla}^a K^i - K_i \widetilde{\nabla}^a \phi^i \right], \tag{20}$$

that corresponds to the integral kernel of a covariant and invariant of gauge symplectic structure defined on the covariant phase space [3].
To finish this section it is important to mention that in the same form, using the previous results we can obtain the results presented in [4]; in this case, we analyze what happens when we add the [GB] topological term to the [DNG] action in string theory, and we found for example, that in the dynamics of deformations exist a contribution non trivial because of the [GB] topological term, therefore, we found a contribution

that does not vanish in the symplectic structure constructed on the covariant phase space for the [DNG-GB] system in string theory. These important results allowed us find using a weakly covariant formalism [10], the canonical variables for [DNG] p-branes and [GB] term in string theory [7], however, as we already commented we found some problems to consider the [DNG-GB] complete system. In this manner, in the next section we will use a strongly covariant formalism introduced in [8] and the results presented in [9] for this problem can be clarified, in other words, we will find the canonical variables for [DNG-GB] system in string theory, which is completely unknown in the literature.

III. The canonical variables for DNG system

As we commented, in [7] we found the canonical variables for [DNG] p-branes (that contain the particular case of strings theory) using a weakly covariant formalism [10], with spacetime indices, and the canonical variables for [GB] topological term with worldsheet indices, in this manner, if we consider the [DNG-GB] system we need rewrite the canonical variables of [GB] term with background spacetime indices and to consider a canonical transformation that leaves the symplectic structure in the Darboux form with some new variables, P and Q, say. However, this problem can be clarified using a strongly covariant formalism introduced in [8] as we will see in the next lines.

Using a strongly covariant formalism introduced by Carter [8], it is found that the symplectic structure for [DNG] branes in a curved background is given by [9]

$$\omega = \sigma_0 \int_{\Sigma} \delta(-\sqrt{-\gamma}\eta^{\mu}{}_{\alpha}\xi^{\alpha})d\bar{\Sigma}_{\mu} = \int_{\Sigma} \sqrt{-\gamma}\widetilde{J}^{\mu} d\bar{\Sigma}_{\mu}, \tag{21}$$

where σ_0 is a fixed parameter, $\eta^{\mu}{}_{\alpha}$ is the (first) fundamental tensor, $\sqrt{-\gamma}\widetilde{J}^{\mu} = \delta(-\sigma_0\sqrt{-\gamma}\eta^{\mu}{}_{\alpha}\xi^{\alpha})$, Σ being a (spacelike) Cauchy surface for the configuration of the brane while $d\bar{\Sigma}_{\mu}$ is the surface measure element of Σ, and is normal to Σ and tangent to the world-surface Here δ is identified as exterior derivative on the covariant phase space. The symplectic structure given in (21) is a exact differential form, since it comes from the exterior derivative of a one form and in particular is an identically closed two-form on the phase space. The closeness is equivalent to the Jacoby identity that Poisson brakets satisfy, in a usual Hamiltonian scheme, and the symplectic current is (world surface) covariantly conserved ($\bar{\nabla}_{\mu}\widetilde{J}^{\mu} = 0$), which guarantees that ω is independent on the choice of Σ and, in particular, is Poncaré invariant.

We can rewrite the symplectic structure given in (25) for identifying the canonical variables for [DNG] branes in the next form

$$\omega = \int_{\Sigma} \delta X^{\alpha}\delta\hat{p_{\alpha}}d\Sigma, \tag{22}$$

where $\hat{p_\alpha} = \sqrt{-\gamma}p_\alpha$, and $p_\alpha = \sigma_0\tau_\alpha$, being τ_α a unit timelike vector field. In this manner, Eq. (32) allows us to identify to X^μ and $\hat{p_\alpha}$ as the canonical conjugate variables in this covariant description of the phase space for [DNG] branes in a curved background (in [7] we identified the canonical variables for [DNG] p-branes in a weakly covariant formalism). It is important to mention that the symplectic structure given in equation (21) and the identification of the canonical variables X^μ and $\hat{p_\alpha}$, allows us to find for example, the covariant Poisson brackets, the Poncaré charges and the closeness of the Poincaré algebra [7, 9].

IV. The canonical variables for GB system in string theory

As we know, the Einstein-Hilbert term is characterized for the action

$$S = \sigma_1 \int \sqrt{-\gamma} R d\bar{\Sigma}, \tag{23}$$

where σ_1 is a fixed parameter and R is the scalar curvature of the embedding [5]. Using the deformations formalism given in [8] we can calculate the variation of S obtaining

$$\delta S = 2\sigma_1 \int \sqrt{-\gamma} G^{\gamma\nu} K_{\gamma\nu\mu}\xi^\mu d\bar{\Sigma} + \sigma_1 \int \sqrt{-\gamma}\bar{\nabla}_\mu(-2G^\mu{}_\nu \xi^\nu + \eta^{\alpha\beta}\delta\rho_\alpha{}^\mu{}_\beta - \eta^\alpha{}_\beta \eta^{\mu\tau}\delta\rho_\alpha{}^\beta{}_\tau) d\bar{\Sigma}, \tag{24}$$

where $G^{\gamma\nu}$ is the internal adjusted Ricci tensor, $K_{\gamma\nu\mu}$ is the second fundamental tensor and $\rho_\alpha{}^\mu{}_\beta$ is the frame gauge internal rotation pseudo-tensor or internal connection [8]. In general, the adjusted Ricci tensor does not vanish for a imbedded p-surface, however, in string theory the Ricci tensor vanishes identically. From last equation we can identify the equations of motion for the brane theory given by

$$G^{\gamma\nu} K_{\gamma\nu}{}^\mu = 0, \tag{25}$$

and such as in the Sect. II, the total divergence term of equation (24) is identified as symplectic potential for the theory under study, given by

$$\Psi^\mu = \sigma_1\sqrt{-\gamma}[-2G^\mu{}_\nu \xi^\nu + \eta^{\alpha\beta}\delta\rho_\alpha{}^\mu{}_\beta - \eta^\alpha{}_\beta \eta^{\mu\tau}\delta\rho_\alpha{}^\beta{}_\tau]. \tag{26}$$

If we take the particular case of string theory in equation (25) the adjusted Ricci tensor vanishes, in this manner, if we utilize the standard canonical formalism to quantize this system, we would not find apparently nothing interesting, however, as we can see in [4] using a weakly covariant formalism introduced in [10] we found that the [GB] term in string theory gives a nontrivial contribution on the Witten covariant phase space leading to a completely different quantum field theory. We can see it if we takes the particular case of string theory in Eq. (26) obtaining

$$\Psi^\mu = \sigma_1\sqrt{-\gamma}[\eta^{\alpha\beta}\delta\rho_\alpha{}^\mu{}_\beta - \eta^\alpha{}_\beta \eta^{\mu\tau}\delta\rho_\alpha{}^\beta{}_\tau], \tag{27}$$

in this manner, we can see that the terms of last equation do not vanish. This result allows us to find the canonical variables for [GB] strings.

In order to continue, we need rewrite the internal connection in terms of the (co) vector ρ_μ defined as

$$\rho_\lambda = \rho_{\lambda}{}^{\mu}_{\nu}\varepsilon^{\nu}{}_{\mu}, \quad \rho_{\lambda}{}^{\mu}_{\nu} = \frac{1}{2}\varepsilon^{\mu}{}_{\nu}\rho_\lambda, \tag{28}$$

where $\varepsilon^{\mu\nu} = 2\iota_0^{[\mu}\iota_1^{\nu]}$, being ι_0^μ a timelike unit vector, and ι_1^μ a spacelike one, which constitute an orthonormal tangent (to the world sheet) frame [8]. Thus, considering the last equation, the symplectic potential given in the expression (27) takes the form

$$\Psi^\mu = \sqrt{-\gamma}\varepsilon^{\mu\nu}\delta\rho_\nu, \tag{29}$$

where we have used the frame gauge property of $\rho_{\lambda}{}^{\mu}_{\nu}$ and consequently of ρ_λ [5].

In this manner, we can define a covariant and gauge invariant symplectic structure for [GB] strings as

$$\omega' = \int_\sigma \delta(\sigma_1\sqrt{-\gamma}\varepsilon^{\mu\nu}\delta\rho_\nu)d\bar{\Sigma}_\mu, \tag{30}$$

therefore, from last equation we can identify as well as for [DNG] system the canonical variables for [GB] strings, this is

$$p_\nu = \sigma_1\sqrt{-\gamma}\varepsilon^{\mu}{}_{\nu}\tau_\mu, \quad q^\nu = \rho^\nu. \tag{31}$$

In this manner, we can see that in this case the canonical variables has spacetime indices contrary to [7] that has worldsheet indices. With these results we can treat the complete [DNG-GB] system which is the purpose of the next section and of this paper.

V. The canonical variables for DNG-GB system in string theory

In this section we will study the [DNG-GB] system in string theory. For that, we begin with the action that describe the system under study, this is

$$S = -\sigma_0 \int \sqrt{-\gamma}d\Sigma + \int \sigma_1\sqrt{-\gamma}Rd\Sigma, \tag{32}$$

now, using the deformations formalism given in [8] we take the variation of last equation and considering the particular case of string theory, finding

$$\delta S = \sigma_0 \int \sqrt{-\gamma}K_\mu\xi^\mu d\Sigma + \int \bar{\nabla}_\mu[-\sigma_0\eta^{\mu}{}_{\nu}\xi^\nu + \sigma_1\varepsilon^{\mu\nu}\delta\rho_\nu]d\Sigma, \tag{33}$$

where we can identify the equations of motion given by

$$K^\mu = 0, \tag{34}$$

that corresponds to the equations of motion for [DNG] strings. On the other hand, the total divergence term of Eq. (33) is identified as symplectic potential for the theory under study

$$\Psi^{\mu} = \sqrt{-\gamma}[-\sigma_0 \eta^{\mu}{}_{\nu}\xi^{\nu} + \sigma_1 \varepsilon^{\mu\nu}\delta\rho_{\nu}], \tag{35}$$

Whit the previous results, form last equation we can obtain the covariant and gauge invariant symplectic structure for [DNG-GB] in string theory, this is

$$\omega = \int_{\Sigma} \delta\hat{P}_{\nu} \wedge \delta Q^{\nu} d\bar{\Sigma}, \tag{36}$$

where

$$\hat{P}_{\nu} = \sqrt{-\gamma} p_{\nu}, \quad \text{and} \quad Q^{\nu} = -\frac{\sigma_1}{\sigma_0}\varepsilon^{\nu\alpha}\rho_{\alpha} + X^{\nu}, \tag{37}$$

with $p_{\nu} = \sigma_0 \tau_{\nu}$. Therefore, we can identify to $\hat{P}_{\nu}$ and Q^{ν} as canonical variables for [DNG-GB] system in string theory which is completely unknown in the literature. We can note that the contribution because of [GB] term on the canonical variable Q^{ν} (see the first term of Q^{ν} in Eq.(37)) will be relevant when is calculated the angular momentum of the complete [DNG-GB] system, and will be important in the complete quantum field theory.

It is important to mention that we have choose the canonical momentum for [DNG] strings (see the equation (22)) and [DNG-GB] strings (see equation (37)) in the same form, the reason is that p_{ν} satisfies the mass shell ($p_{\nu}p^{\nu} = \sigma^2$) and as we know of the literature the mass shell is an important condition to quantum level for [DNG] strings, because of the Virasoro operators and the mass shell conditions determinate the masses of the physical states, in this manner, we also hope that such condition will be important when we analyze the spectrum of [DNG-GB] system in string theory, however this we discuss in future works.

To finish this work, is important to see that if we take $\sigma_1 = 0$ in equation (37) we obtain the result given in Eq. (22). However, we hope the choice of the canonical variables made in this paper as first quantization are the best election, because of the canonical momentum for [DNG] and [DNG-GB] strings coincide, thus, with the results of this paper and the treatment that is found in the literature to quantize [DNG] strings we have the necessary elements to quantize [DNG-GB] strings.

V. Conclusions and prospects

As we can see, using the deformations formalism introduced by Carter and a covariant and gauge invariant symplectic structure, we could find the canonical variables for [DNG-GB] system in string theory which

is absent in the literature. Whit this results we have the necessary elements for study the quantization aspects of [DNG-GB] strings, since for this purpose we need the results of this paper and the solutions to the equation of motion (34) that are given in elementary books on string theory. In this manner, we can observe the change on the resulting quantum field theory of the topology of the world surface given by [GB] term, and thus, find the contribution of such term on the results that we find in the literature for [DNG] strings; however, we will discuss this subject in future works.

In addition to this work, we know that the bosonic strings (which is the case of this work) are not the general case to describe the nature and it is necessary to add the supersymmetry, among other things, in order to give a description of the fermionic matter, in this manner, a interesting question may be the inclusion of supersymmetry to the results of this paper to find the quantization bases for [DNG-GB] in superstring theory an thus giving a complete description of the matter, however, we will discuss this in future works.

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